

## Maths 165

### Review Sheet 2

- Find the derivative of the following functions:
  - $f(x) = x^3(x^4 + 3x^2 + 2x)$
  - $g(x) = \frac{x^2 + x}{x^3 - 1}$
  - $f(t) = \sqrt{t^3} - \frac{5}{\sqrt{t}} + \pi^2$ .
- Find the point where the line normal to the curve  $y = x^2$  at the point  $(1, 1)$  intersects the curve again.
- For what values of  $a$  and  $b$  is the line  $6x + y = b$  tangent to the parabola  $y = ax^2$  when  $x = -2$ ?
- A point moves along the number line so that its position at time  $t \geq 0$  is  $s(t) = 2t^3 - 15t^2 + 36t - 10$ .
  - When and where does the particle change direction?
  - When is the particle moving in the positive direction?
- Compute the following limits:
  - $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x \cos(x)}$
  - $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$
- If  $f(x) = \csc(x) - \cot(x)$ , find  $f'(x)$ .
- Give the equation of the line tangent to the graph of  $y = \sin(x) \cos(x)$  at the point  $\left(\frac{\pi}{4}, \frac{1}{2}\right)$ .
- Differentiate the following:
  - $f(x) = \ln \cos x$
  - $g(x) = \frac{\sqrt{x+1}}{\sqrt[3]{3x+1}}$
  - $y = \sqrt{x} \cos^2(3x)$
  - $f(x) = \sin(\cos(x))$
  - $h(x) = \tan^{-1}(e^{2x})$
- Suppose  $f(3) = 4$ ,  $f(1) = 6$ ,  $f'(3) = 2$ ,  $f'(1) = -2$ ,  $g(3) = 1$ ,  $g(1) = -3$ ,  $g'(3) = 5$ , and  $g'(1) = 1$ . If  $v(x) = f(g(x))$ , find  $v'(3)$ .
- At what point does the line tangent to the ellipse  $2x^2 + 7y^2 = 15$  at the point  $(2, 1)$  intersect the x-axis?
- Find the equation of the tangent line to the curve  $\sin(x + y) = x$  at the point  $(0, \pi)$ .
- Show that the curves  $2x^2 + y^2 = 3$  and  $x = y^2$  are orthogonal.
- Given  $y = t^4 - 4t^3 + 2$  is the position of an object at time  $t$ , find the times when the acceleration is zero and the position and velocity of the object at these times.
- If  $y = x^r e^{sx}$ , where  $r$  and  $s$  are constants, find  $y'$ .
- If  $f(x) = \frac{e^x - e^{\frac{x}{2}}}{2}$ , find  $f''(0)$ .
- If  $f(2) = 3$ ,  $f'(2) = 6$ , and  $f'(3) = -2$ , find  $g'(3)$ , where  $g = f^{-1}$ .
- Given that  $f(x) = 2x^3 + 9x - 4$  is one-to-one, find  $g'(7)$ , where  $g = f^{-1}$ .
- Find the inverse of  $f(x) = \frac{2x - 3}{3x + 7}$ .
- Find the exact value of  $\log_5(3) - \log_5(375)$ .
- Solve for  $x$ :  $\ln(x + 1) + \ln(x) = \ln(2)$ .
- Simplify  $\ln\left(\frac{2x^3y}{\sqrt{z}}\right)$ .

22. Solve for  $x$ :  $e^{\log_2(x)} = 2$ .
23. Compute  $\lim_{x \rightarrow 0^+} \ln(\tan(x))$ .
24. Find the exact value of  $\lim_{x \rightarrow \infty} (\ln(2x - 1) - \ln(3x + 6))$ .
25. If  $f(x) = 3^{\sin(x)}$ , find  $f'(x)$ .
26. Use logarithmic differentiation to find the derivative of  $\frac{(x + 1)^4(x - 5)^6}{\sqrt{2x + 1}}$ .