

Math 165

Review Sheet 1

1. Find the domain of the function

$$f(x) = \frac{x^2 - 3}{\sqrt{x^4 - x^3 - 2x^2}}.$$

2. If $f(x) = \frac{x-1}{x-2}$ and $g(x) = \frac{1}{x}$, find $f \circ g$ and give its domain.

3. A box with a square base and no top has a volume of 32 cubic centimeters. Give the surface area of the box as a function of the length of one side of the base.

4. Give two functions f and g such that $(f \circ g)(x) = \sqrt[3]{1 - x^3}$.

5. Given

$$f(x) = \begin{cases} 2 - x, & \text{if } x < -1 \\ x, & \text{if } -1 \leq x < 1 \\ 4, & \text{if } x = 1 \\ 4 - x, & \text{if } x > 1 \end{cases} \quad (1)$$

- a) Sketch the graph of $f(x)$.

- b) Find the value of the following limits:

$$\lim_{x \rightarrow -1^-} f(x); \quad \lim_{x \rightarrow -1^+} f(x); \quad \lim_{x \rightarrow -1} f(x);$$

$$\lim_{x \rightarrow 1^-} f(x); \quad \lim_{x \rightarrow 1^+} f(x); \quad \lim_{x \rightarrow 1} f(x).$$

6. Determine the infinite limit:

$$\lim_{x \rightarrow 3^-} \frac{x(4-x)}{x-3}.$$

7. Find the vertical asymptotes for

$$f(x) = \frac{6x}{x - x^3}.$$

8. Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^2 + 13x + 12}$.

9. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$.

10. Evaluate $\lim_{x \rightarrow 0} 2x^2 \cos\left(\frac{10}{x}\right)$.

11. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x + 1} - x)$.

12. Let

$$f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 0 \\ \frac{x^2 - 4}{x - 2}, & \text{if } 0 < x < 2 \\ x^3 - 4, & \text{if } x \geq 2 \end{cases} \quad (2)$$

- a) Show $f(x)$ is continuous at $x = 2$ or explain why it is discontinuous.

- b) Show $f(x)$ is continuous at $x = 0$ or explain why it is discontinuous.

13. Prove there is a solution to the equation $x^3 + 2x + 1 = 0$.

14. Given

$$f(x) = \begin{cases} 3x^2 - 2x + 2a, & \text{if } x \leq 1 \\ 3ax - 1, & \text{if } x > 1 \end{cases} \quad (3)$$

- find the value of a that make $f(x)$ continuous everywhere.

15. Draw the graph of a function such that 1) f is discontinuous at $x = a$ (limit DNE), 2) f is discontinuous at $x = d$ ($f(x)$ is not defined), and 3) f is discontinuous at $x = e$ (limit $\neq f(e)$).

16. Refer to the graph given above. Determine where $f(x)$ is not differentiable.

17. A ball is thrown into the air with a speed of 40 feet per second. The height of the ball after t seconds is given by $h(t) = 40t - 16t^2$.

- a) Find the average velocity of the ball from $t = 2$ to $t = 2.5$.

- b) Find the instantaneous velocity of the ball at $t = 2$ seconds.

18. Given $f(x) = \sqrt{x+1}$,

- a) Find the slope of the secant line joining the points $(2, \sqrt{3})$ and $(3, 2)$.

- b) Using the limit definition, find the slope of the tangent line to the graph of $f(x)$ at the point $(3, 2)$. What is the equation of the tangent line?

19. Compute the derivative of $f(x) = \frac{x}{x+1}$ using the definition of the derivative.

20. Use the ϵ, δ definition to show that $\lim_{x \rightarrow 4} (5x + 4) = 24$ and illustrate it with a graph indicating the roles of ϵ and δ .