Homework 2 solutions

§12.4 #16 The figure shows a vector $\vec{a}$ in the xy-plane and a vector $\vec{b}$ in the direction of $\vec{k}$. Their lengths are $|\vec{a}| = 3$ and $|\vec{b}| = 2$.
a) Find $|\vec{a} \times \vec{b}|$
b) Use the right-hand rule to decide whether the components of $\vec{a} \times \vec{b}$ are positive, negative, or 0.

$$a) \quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta = 3 * 2 * \sin(\pi/2) = 6$$

b) $|\vec{a} \times \vec{b}|$ is orthogonal to $\vec{k}$, so it lies in the xy-plane, and its z-coordinate is 0. By the right-hand rule, its y-component is negative and its x-component is positive.

§12.4 #20 Find two unit vectors orthogonal to both $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} + \vec{k}$.

We know that the cross product of two vectors is orthogonal to both. So we calculate

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} * \vec{i} - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} * \vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} * \vec{k} = \vec{i} + \vec{j} - 2\vec{k}.$$

Hence the required **UNIT** vectors are $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{\vec{i}+\vec{j}-2\vec{k}}{\sqrt{6}}$ and $-\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-\vec{i}-\vec{j}+2\vec{k}}{\sqrt{6}}$.

§12.5 #20 Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they are intersecting, find the point of intersection.
$L_1 : x = 1 + 2t, y = 3t, z = 2 - t$
$L_2 : x = -1 + s, y = 4 + s, z = 1 + 3s$

The lines aren't parallel since the direction vectors $\langle 2, 3, -1\rangle$ and $\langle 1, 1, 3\rangle$ aren't parallel. For the lines to intersect we must be able to find one value of t and one value of s that produce the same point from the respective parametric equations.

Thus we need to satisfy the following three equations

$$1 + 2t = -1 + s \qquad 3t = 4 + s \qquad 2 - t = 1 + 3s.$$

Solving the first two equations we get t=6, s=14 and checking, we see that these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.

§12.5 # 35 Find an equation of the plane that passes through the point (6,0,-2) and contains the line x=4-2t, y=3+5t, z=7+4t.

If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\vec{a} = \langle -2, 5, 4\rangle$ is one vector in the plane. We can verify that the given point (6,0,-2) does not lie on this line, so to find another nonparallel vector $\vec{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put t=0, we see that (4,3,7) is on the line, so $\vec{b} = \langle 6 - 4, 0 - 3, -2 - 7\rangle = \langle 2, -3, -9\rangle$ and $\vec{n} = \vec{a} \times \vec{b} = \langle -45 + 12, 8 - 18, 6 - 10\rangle = \langle -33, -10, -4\rangle$. Thus, an equation of the plane is $-33(x - 6) - 10(y - 0) - 4[z - (-2)] = 0$ or $33x + 10y + 4z = 190$.

§12.6 #21-28 Match the equation with its graph (labeled I-VIII). Give **Reasons** for your choices.

There are many ways to solve this set of problems. You could look at the traces of each equation by letting $y = k$. This will immediately eliminate several choices for each equation. By doing this again to the x and z variables, you will know exactly what the traces in each of your three chosen planes are and can choose appropriately from there. The correct choices are
21. VII 22. IV 23. II 24. III 25. VI 26. I 27. VIII 28. V