Homework 2 solutions

§2.3 # 2

a)$\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2+0 = 2$

b)$\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

c)$\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$

d)For $x \approx -1$ and $x > -1$, we see that $f(x) \approx -1$ and $g(x) \approx 0$ with $g(x) > 0$, giving $\frac{f(x)}{g(x)}$ very large and negative. Hence the right hand limit is $\lim_{x\to -1^+} \frac{f(x)}{g(x)} = -\infty$.

For $x \approx -1$ and $x < -1$, we see that $f(x) \approx -1$ and $g(x) \approx 0$ with $g(x) < 0$, giving $\frac{f(x)}{g(x)}$ very large and positive. Hence the left hand limit is $\lim_{x\to -1^-} \frac{f(x)}{g(x)} = +\infty$.

Since these limits disagree, $\lim_{x\to -1} \frac{f(x)}{g(x)}$ limit does not exist.

e)$\lim_{x\to 2} x^3 f(x) = [\lim_{x\to 2} x^3][\lim_{x\to 2} f(x)] = 2^3 \cdot 2 = 16$

f)$\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

§2.3 # 6

$$\begin{aligned}
\lim_{x\to -2} \sqrt{u^4+3u+6} &= \sqrt{\lim_{x\to -2}(u^4+3u+6)} \text{ (by 11)} \\
&= \sqrt{\lim_{x\to -2} u^4 + 3\lim_{x\to -2} u + \lim_{x\to -2} 6} \text{ (by 1, 3)} \\
&= \sqrt{(-2)^4+3(-2)+6} = \sqrt{16-6+6} = \sqrt{16} = 4 \text{ (by 9, 8, 7)}
\end{aligned}$$

Note: The last step can also be justified by using just the Direct Substitution Property, which we called (12) in class.

§2.3 # 21

$$\begin{aligned}
\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h} &= \lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\cdot\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\\
&= \lim_{h\to 0}\frac{h}{h(\sqrt{9+h}+3)}\\
&= \lim_{h\to 0}\frac{1}{\sqrt{9+h}+3}\\
&= \frac{1}{\sqrt{9+0}+3} = \frac{1}{3+3} = \frac{1}{6}.
\end{aligned}$$

§2.3 # 43 Evaluate

$$\lim_{x\to 0.5^-}\frac{2x-1}{|2x^3-x^2|}.$$

Note that $|2x^3-x^2| = |x^2(2x-1)| = |x^2|\cdot|2x-1| = x^2|2x-1|$ and

$$|2x-1| = \begin{cases} 2x-1, & \text{if } 2x-1\geq 0, \text{ i.e. } x\geq 0.5\\ -(2x-1), & \text{if } 2x-1<0, \text{ i.e. } x<0.5\end{cases} \tag{1}$$

Thus we have that $|2x^3-x^2| = x^2[-(2x-1)]$ for $x<0.5$ and so

$$\lim_{x\to 0.5^-}\frac{2x-1}{|2x^3-x^2|} = \lim_{x\to 0.5^-}\frac{2x-1}{x^2[-(2x-1)]} = \lim_{x\to 0.5^-}\frac{-1}{x^2} = \frac{-1}{(0.5)^2} = \frac{-1}{0.25} = -4.$$

§2.4 #2 Use the given graph of f to find a number δ such that $0<|x-3|<\delta$ then $|f(x)-2|<0.5$. If $|f(x)-2|<0.5$, then $-0.5<f(x)-2<0.5$, i.e., $1.5<f(x)<2.5$. From the graph, we see that the last inequality is true if $2.6<x<3.8$, so we can take $\delta = \min[3-2.6, 3.8-3] = \min[0.4, 0.8] = 0.4$

§2.4 # 12 Let $T(w) = 0.1w^2 + 2.155w + 20$.

a) How much power is needed to maintain the temperature at 200^oC?

$T(w) = 200 \Rightarrow w \approx 33.0$ watts by the quadratic formula.

b) If the temp. is allowed to vary from 200^oC by up to $\pm 1^o$C, what range in wattage is allowed for the input?

Using the quadratic formula again we see $199 \leq T \leq 201 \Rightarrow 32.89 < w < 33.11$. (Answers might vary slightly from rounding error from your calculator.)

c) In this problem, x is the input power, $f(x)$ is the temperature, a is the target input power given in part (a), L is the target temperature (200^oC), ϵ is the tolerance in the temperature (1^oC), and δ is the tolerance in the power input in watts indicated in part (b) (0.11 watts).

§2.5 # 45 For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x, & \text{if x} < 2 \\ x^3 - cx, & \text{if x} \geq 2 \end{cases} \tag{2}$$

For any c value, the two functions which make up f are continuous on $(-\infty, 2)$ and $(2, +\infty)$ individually. So we only need to make sure that f is continuous at $x = 2$.

Now

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} cx^2 + 2x = 4c + 4$$

and

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^3 - cx = 8 - 2c = f(2).$$

So, f is continuous at $x = 2$ when $4c + 4 = 8 - 2c$, i.e., $c = \frac{2}{3}$.

Thus for f to be continuous on $(-\infty, \infty)$, we must have $c = \frac{2}{3}$.